Technical Comments

Comment on "Simplified Solutions for Ablation in a Finite Slab"

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BECAUSE of the topical importance of charring ablators and because of the extensive computer calculations generally required in heat-shield design, any approximate solutions of charring ablator behavior are of much interest. For the same reasons, it seems worthwhile to examine more closely a recent note by Chen[1] which presents some solutions pertaining to a simplified model of ablation in a finite slab.

In the process of supposedly obtaining some analytical solutions, the author makes many simplifying assumptions. It is perhaps best to emphasize at the outset that this writer is in complete accord with Chen's philosophy of choosing a highly idealized model to facilitate solution, and no approximation is claimed here as being invalid per se. However, one explicit assumption made by the author actually contradicts another one (see item 1 below).

There are also some other points on which specific comment seems desirable as discussed below. The nomenclature is that of the original paper. Figure 1 is an exact copy of Fig. 1 of the original paper, and defines the various "layers" referred to.

1) To simplify the solution of the energy equation in the "char-gas layer," Chen assumes that temperature gradients in the virgin material are negligible. While this assumption is not necessarily wrong in itself, it is contradicted by the assumption of a constant back-face temperature. The temperature in the "virgin-material layer" drops from T_m to T_i over a distance δ_s. As δ_s shrinks with time with the temperature difference staying constant, it is obvious that gradients must eventually become large. More strictly, as may be shown by a simple inequality analysis, for Chen's assumption to be valid, it is necessary that throughout the period of ablation

$$(T_m - T_i)/(T_0 - T_m) \ll (k_c/k_s)(\delta_s/\delta_c)$$

This condition will hold in realistic situations only for $(T_0 - T_m) \gg (T_m - T_i)$ or $T_i \approx T_m$, in which case a solution for temperature distribution in the virgin material seems superfluous.

2) Chen's statement of the energy balance at the char-virgin interface, expressed by his Eq. 10, is incorrect. This energy balance should be derived thus:

a) (Heat flux conducted to interface from char) − (Heat flux conducted to virgin material) = (L) × (Mass rate of charring of virgin material per unit area).

b) (Mass rate of charring) = (ρ_s) × (Velocity of interface relative to stationary observer).

c) (Velocity of interface) = $(ds/d\theta) + (d\delta_c/d\theta)$

d) Hence Eq. 10, which is essentially statement a), should read: at $\xi = \delta_c$,

$$k_s(\partial T_s/\partial \xi) - k_c(\partial T_c/\partial \xi) = L\rho_s[(ds/d\theta) + (d\delta_c/d\theta)]$$

Equation (11) also should be modified similarly. In the general case, δ_c changes with time (as it must if any char is to develop); therefore it is not permissible to neglect $d\delta_c/d\theta$. However, in the special asymptotic case of quasi-steady ablation, δ_c is a constant and Chen's neglect of the $d\delta_c/d\theta$ term is valid. The resulting solution for temperature distribution in the char is therefore correct for this one case. However, it should be pointed out that this solution is a limiting case of a very much more general quasi-steady solution derived by Barriault and Yos in 1960.[2]

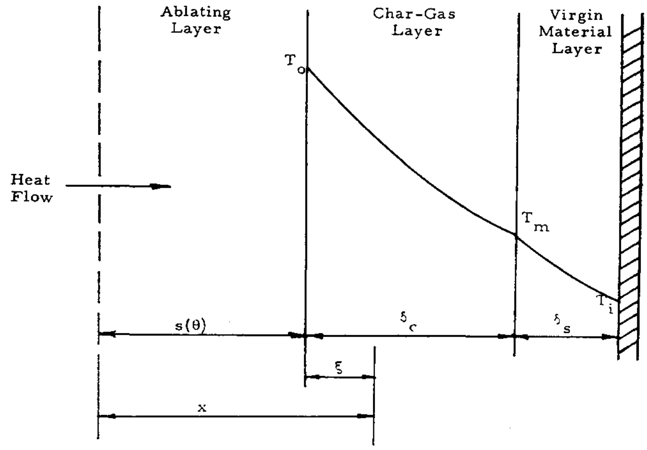

Fig. 1 Ablation model.

3) As regards the transient temperature distribution in the "char-gas layer", Chen's solution is valid (apart from a minor modification necessitated by the correction of Eq. 10) only if the following two unstated conditions are satisfied.

a) The "ablating-layer" thickness s is always equal to the char-gas-layer thickness δ_c. This is a necessary precondition for Eq. (22) to satisfy boundary condition (6). In no way can this relationship emerge from the solution (see remarks below under item 5).

b) The surface heat input $h(T_g - T_0)$ assumed constant for the quasi-steady state is now proportional to $\theta^{-1/2}$. This is necessary according to Eq. (23), if λ is to be independent of θ as assumed by Chen. As h is constant (Assumption 8) and as T_0 must be constant [Eq. (24)], it follows that T_g must have the singular form

$$T_g = T_0 + C\theta^{-1/2}$$

where C is a constant. It is difficult to know, in the absence of any statement, whether the author intended these assumptions to be implied. For, the unrealistic, arbitrary, and drastic limitations they impose on the generality of the solution raise serious doubts as to the meaningfulness of this solution.

4) In the solution of the virgin-zone temperature distribution for both the transient and quasi-steady cases, Chen uses the following equations:

$$\partial T/\partial \theta = \alpha(\partial^2 T/\partial x^2) \quad (1)$$

$$T_s(\delta_c) = T_m \quad (7)$$

$$T_s(\delta_s) = T_i \quad (8)$$

Equation (1) is in the coordinates of a stationary observer; boundary condition (7) is in the coordinates of an observer moving with the char surface; boundary condition (8) is in the coordinates of an observer moving with the char-virgin interface (see Fig. 1). Chen solves these equations formally, as they stand, without any transformations to bring them to the same set of coordinates. Understandably, the resulting solutions are both incorrect. For example, in the quasi-steady solution, $\partial T/\partial \theta$ is equated to zero in Eq. (1) (the fact

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that time derivatives are now zero only for an observer moving with the char is forgotten), and solution of the resulting equation with boundary conditions (7) and (8) leads Chen to deduce a linear variation of temperature in the virgin material!

5) The model of ablation presented considers both a charring process and a surface-mass removal process. However, Chen does not explicitly make any statement regarding the mathematical model for surface-mass loss. It should be pointed out that there are four unknowns in the problem: a) the temperature distribution in the char-gas layer, b) the temperature distribution in the virgin-material layer, c) the thickness of the ablating layer s, representing the amount of surface matter removed, and d) the thickness of the char-gas layer δ_c. (Of course, once s and δ_c are known, the thickness of the virgin-material layer δ_s automatically follows.)

Unknowns a) and b) each are described by a partial differential equation, two boundary conditions, and an initial condition. Unknowns c) and d) each have an initial condition (presumably, $s(0) = \delta_c(0) = 0$). However, they have only one equation between them, viz., Eq. (10). It is therefore necessary for the solution of this problem to have another equation involving s and/or δ_c. For the steady-state case, such an equation is offered by the implicit relation:

$$(d\delta_c/d\theta) = 0$$

For the transient case, such a relationship is also necessary. This could be provided by the (apparent) assumption mentioned under item 3 preceding,

$$s(\theta) = \delta_c(\theta)$$

This relationship does not seem to have any justification, either theoretical or experimental. However, without this assumption, Chen's stated solution would not be a solution at all.

References

[1] Chen, N. H., "Simplified solutions for ablation in a finite slab," AIAA J. **3**, 1148–1149 (1965).

[2] Barriault, R. J. and Yos, J., "Analysis of the ablation of plastic heat shields that form a charred surface layer," ARS J. **30**, 823–829 (1960).

Reply by Author to D. B. Adarkar

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ATTENTION should be called to the fact that ablation is a complicated phenomenon. It deals with heat transfer (conduction, convection, and radiation), mass transfer (diffusion), chemical reaction (combustion), and kinetics. The products of the combustion are char and multicomponent gaseous mixtures. In addition, the heat input from the environment to initiate the ablation in a rocket motor also is concerned with multicomponent exhaust gaseous mixtures of which the composition and the convective heat-transfer coefficient still are evaluated approximately. Because of this complex and approximate nature, it is definitely impossible to obtain a closed form solution to account for all these effects. Hence, for the past years, a great number of articles (probably over a hundred) have appeared on the basis of simplified assumptions.

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With these simplified assumptions in mind, it is evident that the solution is not exact but approximate. The purpose of my previous note[1] is to obtain a simple and approximate analytical expression with simplified assumptions.

To fulfill this intention, the temperature gradient in the virgin material layer was assumed negligible in order to reduce the boundary condition, Eq. (10), to a simpler form, Eq. (11). Consequently, the final solution would be simpler. The validity of this assumption easily can be seen by the fact that, for a thin virgin-material layer, the resistance to the heat flow is comparatively small, so that the temperature gradient cannot be great. It is always desired to have economical but adequate thickness of ablative material as insulator in a rocket motor. This implies that the insulation should be designed as thin as possible, such that a large part of the insulation will undergo ablation to absorb a large part of the heat from the environment. Only a small portion of the heat transfers to the thin layer of the virgin material. Furthermore, the firing of a rocket motor is usually short. For all these reasons, it appears that this assumption of negligible temperature gradient in the virgin material layer is very reasonable for obtaining simplified analytical solutions.

To repeat, the prime objective of my note is to acquire an analytical solution through simplified assumptions. In order to save space in the publication, no attention has been paid to the consistency of the formulation of the boundary conditions and the governing partial differential equations. As long as the idea is understood, the objective is accomplished. The use of the coordinate x in Eq. (4) means that the initial temperature in the char layer and the virgin-material layer before ablation is T_i. Its use appears to be more general. The use of Eq. (1) to represent the virgin-material layer also is clear. For saving space, it is unnecessary to change coordinate, provided that the correct values and boundary conditions are applied.

It is incorrect to write the expression

$$L\rho_s[(ds/d\theta) + (d\delta_c/d\theta)]$$

as indicated by Adarkar. The term δ_c is not the dependent variable. The $ds/d\theta$ term applies not only to the ablating layer, but also to the char-gas layer as well. Hence, this term is sufficient, and the term $d\delta_c/d\theta$ is wrong. Even if the last term is changed to $d\xi/d\theta$, it is still unnecessary to have it. Hence, the formulation of Eq. (10) and Eq. (11) is still correct inasmuch as the rate of the heat penetration is still $ds/d\theta$ and the temperature gradient in the virgin-material layer is assumed negligible.

Equation (22) does satisfy the boundary condition Eq. (6) because, at δ_c, Eq. (19) really becomes (but not the unlikely case as indicated by Adarkar)

$$s = \delta_c = 2\lambda(\alpha\theta)^{1/2}$$

Equation (23) comes from Eq. (22) and Eq. (9). The latter is the boundary condition at which the heat flows from the environment. Some investigators have pointed out that the heat flux as $H(\theta)$ could be a function of time. For this reason, this $H(\theta)$ is equal to $h(T_g - T_0)$ in my note. In order to have approximate solution in our problem, this $h(T_g - T_0)$ also will be proportional to $1/\theta^{1/2}$, so that the time term can be cancelled out. For the sake of saving space, this explanation did not appear in my note because it is already understood by those who have had sufficient knowledge on ablation.

It is now necessary to explain the proposed model and the formulation. The proposed model represents the ablation with the removal of the ablating layer. Because of this removal, in order to calculate the temperature gradient in the char-gas layer, it is proposed to transform the independent variable x in Eq. (1) to another independent variable ξ by Eq. (2). However, in the calculation of the temperature in the virgin-material layer, it is unnecessary to make this